

Resistivity and $1/f$ Noise in Non-Metallic Phase Separated Manganites

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(February 1, 2008)

A simple model is proposed to calculate resistivity, magnetoresistance, and noise spectrum in non-metallic phase-separated manganites containing small metallic droplets (magnetic polarons). The system is taken to be far from the percolation transition into a metallic state. It is assumed that the charge transfer occurs due to electron tunneling from one droplet to another through the insulating medium. As a result of this tunneling, the droplets acquire or lose extra electrons forming metastable two-electron and empty states. In the framework of this model, explicit expressions for dc conductivity and noise power of the system are derived. It is shown that the noise spectrum has $1/f$ form in the low-frequency range.

PACS numbers: 75.30.Vn, 72.70.+m, 72.20.Jv

I. INTRODUCTION

Recent experimental [1–6] and theoretical [7–12] papers provide a strong evidence for the existence of the phase separated state in the perovskite manganites with the colossal magnetoresistance (CMR). Both experiment and theory demonstrate that under certain conditions the material in the phase separated state consists of small ferromagnetic metallic droplets, or magnetic polarons, embedded into the insulating antiferromagnetic (AFM) matrix. The droplet in the ground state contains one charge carrier (electron or hole) confined in a potential well of ferromagnetically ordered local spins. In Refs. [11,12] the model allowing the estimation of the droplet radius and droplet concentration was proposed. According to this model, the radius of the droplet a is defined from the minimization of the energy: $E \sim t(\pi d/a)^2 + JS^2(4\pi/3)(a/d)^3$, where the first term is related to the kinetic energy of electron delocalization in the bubble with the radius a , and the second term corresponds to the loss in the energy of the Heisenberg AFM exchange due to ferromagnetic ordering of local spins S inside the bubble. The minimization of the energy with respect to droplet radius yields the following estimate for a in 3D case: $(a/d) \sim (t/JS^2)^{1/5}$, where J is the AFM Heisenberg exchange, t is the bandwidth, and d

is the intersite distance. The number of charge carriers is proportional to the electron (hole) doping x . Volume concentration of metallic droplets increases with x and with the decrease of the temperature since the droplet radius a decreases with temperature growth. As a result, at some critical concentration of holes $x_c \sim (d/a)^3$ the droplets start to overlap and the percolation metal-insulator transition occurs in the system [8]. In Ref. [6] it was observed that heating and cooling of the perovskite manganites in the phase separated state is accompanied by strong hysteresis in the magnetization and the resistance. In addition, the giant $1/f$ noise was measured in these experiments. The noise power is very high even far from the percolation threshold and drastically increases in its vicinity. The noise spectrum is close to $1/f$ form in the 1 – 1000 Hz frequency range.

In this paper, we calculate conductivity, magnetoresistance, and noise spectral power of the system in the phase separated state. The calculations are based on the results of Refs. [11,12] and on a simple model for the tunneling conductivity of the material accounting for electron jumps from one polaron to another. The concentration range not too close to the percolation transition is considered.

We use the terms (*magnetic*) *polarons* and *droplets* interchangeably throughout the paper.

II. CONDUCTIVITY

Let us consider an insulating antiferromagnetic sample of volume V_s in electric field \mathbf{E} . The total number of magnetic polarons in the volume is N , and thus their spatial density is $n = N/V_s$. As it was mentioned before, the number of polarons is assumed to be equal to the number of charge carriers introduced by doping. Neglecting the conductivity of the insulating phase, we assume that charge carriers are only located within the droplets. The charge transfer can thus occur either due to the motion of the droplets or due to the electron tunneling. The former mechanism is less effective: Indeed, the motion of a droplet is accompanied by a considerable rearrangement of the local magnetic structure, which results in the big effective mass of magnetic polarons. In addition, the droplets are expected to be easily pinned by crystal

lattice defects. Thus, it is realistic to assume that the charge transport is essentially due to electron transitions between the droplets.

A magnetic polaron in the ground state contains one electron. As a result of a tunneling process, droplets with more than one electron are created, and some droplets become empty (the lifetime of such excitations is discussed in the end of this Section). If the energy of an empty droplet $E(0)$ is taken to be zero, the energy of a droplet with one electron can be estimated as $E(1) \sim t(d/a)^2$. This is essentially the kinetic energy of an electron localized in the sphere of the radius a . In the same way, the energy of two-electron magnetic polaron $E(2) \sim 2E(1) + U$, with U being the interaction energy of the two electrons. In all these estimates, we have disregarded the surface energy, which is expected to be small [11]. Thus, $E(2) + E(0) > 2E(1)$, and the creation of two-electron droplets is associated with the energy barrier of the order of $A \equiv E(2) - 2E(1) \sim U$. It is clear that the interaction energy U of two electrons in one droplet is determined mainly by the Coulomb repulsion of these electrons, hence $A \sim e^2/\epsilon a$, where ϵ is the static dielectric constant, which in real manganites can be rather large ($\epsilon \sim 20$). We assume below that the mean distance between the droplets is $n^{-1/3} \gg a$ (the droplets do not overlap). Then, A is larger than the average Coulomb energy $e^2 n^{1/3}/\epsilon$. Since the characteristic value of the droplet radius a is of the order of 10 Å [11,12], we have $A/k_B \sim 1000K$ and $A > k_B T$ in the case under study. In the following, we assume that the temperature is low, $A \gg k_B T$, and we do not consider a possibility of the formation of the droplets with three or more electrons. Even in the case when these excitations are stable, it can be shown that far from the percolation threshold the strong Coulomb interaction suppresses their contribution to the conductivity (giving rise only to the next order terms with respect to $\exp(-A/k_B T)$).

Let us denote the numbers of single-electron, two-electron, and empty droplets as N_1 , N_2 , and N_3 , respectively. According to our model, $N_2 = N_3$, $N_1 + 2N_2 = N$, and N is constant. Before turning to conductivity, we evaluate the thermal averages of N_1 and N_2 . To this end, we note that the number P_N^m of possible states with m two-electron droplets and m empty droplets equals $C_N^m C_{N-m}^m$, with C_N^m being the binomial coefficients. Since the created pairs of droplets are independent, we write the partition function in the form

$$Z = \sum_{m=0}^{N/2} P_N^m \exp(-m\beta), \quad \beta = A/k_B T. \quad (1)$$

Though the sum can be evaluated exactly and expressed in terms of the Legendre polynomials for arbitrary N , it is more convenient to use the Stirling formula for the factorials and the condition that the sample is macroscopic, $N \gg 1$. Approximating the sum by an integral,

$$Z = \int_0^{N/2} dm \times \exp \left[-m\beta - N \ln \left(1 - \frac{2m}{N} \right) + 2m \ln \left(\frac{N}{m} - 2 \right) \right],$$

calculating it in the saddle-point approximation, and subsequently evaluating in the same way the statistical average of N_2 ,

$$\bar{N}_2 = Z^{-1} \sum_{m=0}^{N/2} m P_N^m \exp(-m\beta) = -\frac{\partial}{\partial \beta} \ln Z, \quad (2)$$

we easily obtain

$$\begin{aligned} \bar{N}_2 &= N \exp(-A/2k_B T), \\ \bar{N}_1 &= N - 2\bar{N}_2 = N [1 - 2\exp(-A/2k_B T)]. \end{aligned} \quad (3)$$

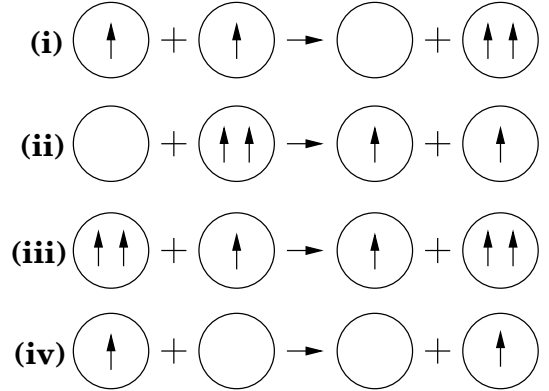


FIG. 1. Elementary tunneling processes.

Now we calculate the conductivity. Within the framework of the proposed model the electron tunneling occurs via one of the four following processes illustrated in Fig. 1,

- (i) In the initial state we have two droplets in the ground state, and after tunneling in the final state we have an empty droplet and a droplet with two electrons;
- (ii) An empty droplet and a two-electron droplet in the initial state transform into two droplets in the ground state (two droplets with one electron);
- (iii) A two-electron droplet and a single-electron droplet exchange their positions by transferring an electron from one droplet to the other;
- (iv) An empty droplet and a single-electron droplet exchange their positions by transferring an electron from one droplet to the other.

In the linear regime, all these processes contribute to the current density j independently, $j = j_1 + j_2 + j_3 + j_4$. The contributions of the first two processes read

$$j_{1,2} = en_{1,2} \left\langle \sum_i v_{1,2}^i \right\rangle, \quad (4)$$

where $n_{1,2} = N_{1,2}/V_s$ are the densities of the single- and two-electron droplets, and $\langle \dots \rangle$ stands for statistical and time averages. The appearance of the factors $n_{1,2}$ reflects the fact that the electron tunnels *from* a single-electron droplet (process (i)) or two-electron (ii) droplet. The summation in Eq. (4) is performed over all magnetic polarons the electron can tunnel *to* — one-electron droplets for the process (i) and empty droplets for the process (ii). Finally, the components of average electron velocity $\langle v_{1,2}^i \rangle$ along the direction of the electric field are obviously found as [13]

$$\left\langle \sum_i v_{1,2}^i \right\rangle = \left\langle \sum_i \frac{r^i \cos \theta^i}{\tau_{1,2}(r^i, \theta^i)} \right\rangle, \quad (5)$$

where r^i and θ^i are the electron tunneling length (the distance between the droplets) and the angle between the electric field and the direction of motion, respectively, and $\tau_{1,2}(r^i, \theta^i)$ are characteristic times associated with the tunneling processes. The relation between $\tau_1(r, \theta)$ and $\tau_2(r, \theta)$ can be found from the following considerations. Near the equilibrium, the number of two-electron droplets, excited per unit time, equals to the number of the decaying two-electron droplets. We thus have the detail balance relation,

$$\frac{\bar{N}_1^2}{\tau_1(r, \theta)} = \frac{\bar{N}_2^2}{\tau_2(r, \theta)}, \quad (6)$$

where we have taken into account that the probability of the formation of a two-electron droplet is proportional to the total number N_1 of the single-electron states multiplied by the number of available hopping destinations, which also equals N_1 . Similarly, the probability of decay of a two-electron droplet is proportional to $N_2 N_3 = N_2^2$. Eq. (6) implies $\tau_2(r, \theta) = \tau_1(r, \theta) \exp(-A/k_B T)$. We write then the conventional expression for the tunneling times [13] in the following form,

$$\tau_{1,2}(r, \theta) = \omega_0^{-1} \exp \left(\frac{r}{l} \pm \frac{A}{2k_B T} - \frac{eEr \cos \theta}{k_B T} \right), \quad (7)$$

where l and ω_0 are the characteristic tunneling length and magnon frequency, and we have taken into account the contribution of the external electric field to the tunneling probability.

To perform the averaging, we assume that the centers of the magnetic polarons are randomly positioned in space and the average distance $n^{-1/3}$ between them is much larger than the droplet radius a . Both assumptions seem to be perfectly justified far below the percolation threshold. Then the averaged sum in Eq. (4) is essentially the space average of v^i , multiplied by the number of droplets available for hopping (N_1 for the process (i) and

N_2 for the process (ii)). Expanding in $eEl/k_B T \ll 1$, we obtain

$$\begin{aligned} \left\langle \sum_i v_{1,2}^i \right\rangle &= \frac{eE\omega_0}{k_B T} N_{1,2} e^{-A/2k_B T} \left\langle r^2 \cos^2 \theta e^{-r/l} \right\rangle_V, \\ \langle \dots \rangle_V &= V_s^{-1} \int \dots d^3 \mathbf{r}. \end{aligned} \quad (8)$$

In Eq. (8), electric field is outside the averaging. Rigorously speaking, this means that the characteristic hopping length l is larger than the interdroplet distance $n^{-1/3}$ and our approach is valid only when the droplet concentration is not too small.

Substituting Eq. (8) into Eq. (4) and performing the integration, we find

$$j_{1,2} = \frac{32\pi e^2 E \omega_0 l^5 n_{1,2}^2}{k_B T} \exp(-A/2k_B T). \quad (9)$$

In the processes (iii) and (iv) the free energy of the system is not changed after the tunneling, and we write the characteristic times as

$$\tau_{3,4}(r, \theta) = \omega_0^{-1} \exp(r/l - eEr \cos \theta / k_B T). \quad (10)$$

The contribution of these two processes to the current is calculated similarly to that of (i) and (ii). For the process (iii) the number of magnetic polarons from which the electron may tunnel, is N_2 , whereas the number of accepting droplets is N_1 . In the same way, for the process (iv) these numbers are N_1 and $N_3 = N_2$, respectively. Consequently, the factors $n_{1,2}^2$ in Eq. (9) are replaced by $n_1 n_2$,

$$j_{3,4} = \frac{32\pi e^2 E \omega_0 l^5 n_1 n_2}{k_B T}. \quad (11)$$

From Eqs. (9) and (11) we now obtain the dc conductivity $\sigma = j/E$,

$$\sigma = \frac{32\pi e^2 \omega_0 l^5}{k_B T} \left[2n_1 n_2 + n_1^2 e^{-A/2k_B T} + n_2^2 e^{A/2k_B T} \right]. \quad (12)$$

In this Section we are only interested in the average conductivity; fluctuations lead to the appearance of noise and are considered in Section IV. Using Eq. (3), we find that all the four processes illustrated in Fig. 1 give identical contributions to the conductivity; for $A \gg k_B T$ the average conductivity (for which we retain the notation σ) reads

$$\sigma = \frac{128\pi e^2 n^2 \omega_0 l^5}{k_B T} \exp(-A/2k_B T). \quad (13)$$

We see that the conductivity increases with temperature as $\sigma(T) \propto T^{-1} \exp(-A/2k_B T)$, which is typical for tunneling systems (see *e.g.* Ref. [13]).

At this point, let us discuss the applicability range of our model. The essence of our picture is the existence of different types of droplets. Of course, only single-electron droplets are stable. Obviously, an empty droplet decays during the time of the order of $1/\omega_0$. On the other hand, following the above discussion, the empty droplet should acquire an electron from neighboring one-electron or two-electron droplets during the characteristic time τ_0 , which can be easily calculated based on the following considerations. The probability P per unit time for an empty droplet to acquire one electron can be written as

$$P = 4\pi\omega_0 \int_0^\infty e^{-r/l} (n_1 + n_2 \exp(A/k_B T)) r^2 dr, \quad (14)$$

where the terms with n_1 and n_2 correspond to the electron transfer from single- and two- electron droplets, respectively. Performing integration in Eq. (14) and using Eq. (3), we find

$$\tau_0 = \frac{\exp(-A/2k_B T)}{8\pi\omega_0 l^3 n} \quad (15)$$

Just the same estimate can be obtained for the characteristic time of electrons leaving two-electron droplets. For our picture with empty and two-electron droplets to be valid, the following condition must be met, $\tau_0 \ll \omega_0^{-1}$. Thus, our approach is valid at sufficiently low temperatures, $k_B T \ll A$, and for not too small droplet density n .

The applicability of our approach also implies that $l > a, n^{-1/3}$. It is of interest to consider also the case of $l \sim a$ and/or low droplet concentrations. In this situation, in usual hopping systems, the conductivity strongly depends on the geometry of current paths [14]. This causes exponential dependence of conductivity on the carrier concentration. However, our system turns out to be more complicated than those commonly invoked for hopping conductivity. It involves different types of hopping centers giving rise to unusual geometry of current paths. Therefore, the conventional approaches used for hopping can not be applied straightforwardly to the analysis of our model at low droplet concentration or at $l \sim a$. Despite these complications, we believe that the expression for the conductivity in the case $l \lesssim n^{-1/3}$ includes the percolation-related factor $\exp(-\beta/n^{1/3}a)$, with β of the order one [14], though currently we have no rigorous proof of this statement. The results below for magnetoresistance and noise are insensitive to this factor, and therefore we expect them to be valid in a general case.

III. MAGNETORESISTANCE

As we already discussed, below the percolation threshold when the volume fraction of droplets $n < n_c$, a typical value of A/k_B is mainly determined by Coulomb interactions between two electrons inside the droplet $A \sim e^2/\epsilon a$

and has a typical value of $1000K$. Now we can use this estimate to analyze the magnetoresistance in non-metallic phase-separated manganites. To do that, we use the expression for the radius of the magnetic polaron, obtained in the introduction $a \propto d(t/J S^2)^{1/5}$. Recall once more that here $J \sim 100$ K is an AFM Heisenberg exchange between the local spins $S = 3/2$. It is natural to conclude that in the magnetic field H the Heisenberg exchange integral J decreases according to the formula $J(H)S^2 = J(0)S^2 - g\mu_B H S$, where μ_B and g are the Bohr magneton and the gyromagnetic ratio, respectively. Consequently, the value of A is decreasing linearly in the experimentally accessible range of magnetic fields, and for the excitation energy we obtain

$$A(H) = A(0)[1 - bH], \quad b = \frac{1}{5} \frac{g\mu_B}{J(0)S}. \quad (16)$$

It follows now from Eq. (13) that the magnetoresistance is negative and for temperatures $T < A/k_B$ reads

$$\begin{aligned} |MR| &\equiv \frac{\rho(0) - \rho(H)}{\rho(H)} = \exp \left[\frac{A(0) - A(H)}{2k_B T} \right] - 1 \\ &= \exp \left(\frac{bHA}{2k_B T} \right) - 1. \end{aligned} \quad (17)$$

For low magnetic fields and not very small temperatures the absolute value of the magnetoresistance is small, $|MR| = bHA/2k_B T \ll 1$. In higher fields (but still $bH \ll 1$) the absolute value of magnetoresistance eventually exceeds one and behaves in the exponential fashion, $|MR| = \exp(bHA/2k_B T)$. Note that for temperatures $T \lesssim A/k_B$ and for typical gyromagnetic ratios $g \sim 10$ the magnetoresistance in our region of doping becomes larger than one by absolute value only in relatively high magnetic fields $H \sim 10$ T.

IV. $1/f$ NOISE POWER

Recently, Podzorov *et al* [6] reported the observation of giant $1/f$ noise in perovskite manganites in the phase separated regime. Generally, systems with distributed hopping lengths are standard objects which exhibit $1/f$ noise (for review, see Refs. [15,16]). The purpose of this Section is to study low-frequency noise within the framework of the model used to calculate the conductivity in Section II, and show that it has, indeed, $1/f$ form.

Starting from the Ohm's law $U = IL/\sigma S$ (where L and S are the sample length and the cross-section, respectively) and assuming that the measuring circuit is stabilized ($I = \text{const}$), we can present the voltage noise at the frequency ω , $\langle \delta U^2 \rangle_\omega$, in the following way,

$$\langle \delta U^2 \rangle_\omega = U_{dc}^2 \frac{\langle \delta \sigma^2 \rangle_\omega}{\sigma^2}, \quad (18)$$

where U_{dc} is the time-averaged voltage, and $\langle \delta \sigma^2 \rangle_\omega$ is the noise spectrum of the fluctuations of the conductivity.

If we disregard possible fluctuations of temperature in the system, the only source of the fluctuations in our model is those of the occupation numbers n_1 and n_2 . Using the conservation law $n_1 + 2n_2 = n$, we find from Eq. (12)

$$\delta\sigma = \sigma \frac{\delta n_2}{\bar{n}_2} [1 - 2 \exp(-A/2k_B T)]. \quad (19)$$

We thus need to find the fluctuation spectrum $\langle \delta n_2^2 \rangle_\omega$. Following the general prescription [17], we recollect that the two-electron droplets decay via the process (ii), and the relaxation equation has the form

$$\delta \dot{n}_2 = -\frac{\delta n_2}{\tau(r)}, \quad \tau(r) = \omega_0^{-1} \exp(r/l - A/2k_B T), \quad (20)$$

where we have neglected the effect of the electric field. The fluctuation spectrum then reads [17]

$$\begin{aligned} \langle \delta n_2^2 \rangle_\omega &= \langle \delta n_2^2 \rangle_T \left\langle \sum_i \frac{2\tau(r^i)}{1 + \omega^2 \tau^2(r^i)} \right\rangle \\ &= 8\pi \bar{n}_2 \langle \delta n_2^2 \rangle_T \int_0^\infty \frac{\tau(r)}{1 + \omega^2 \tau^2(r)} r^2 dr, \end{aligned} \quad (21)$$

where $\langle \delta n_2^2 \rangle_T$ is the thermal average of the variation of n_2 , and the summation is performed over the pairs "empty droplet – two-electron droplet", with r_i being the distance between the sites in a pair. The average in Eq. (21) is essentially a spatial integral, with the main contribution coming from short distances,

$$\langle \delta n_2^2 \rangle_\omega = 8\pi \bar{n}_2 \langle \delta n_2^2 \rangle_T \int_0^\infty \frac{\tau(r)}{1 + \omega^2 \tau^2(r)} r^2 dr. \quad (22)$$

Note that Eq.(22) is valid for an arbitrary relation between l and a , not necessarily for $a \ll l$.

We are interested below in the frequency range

$$\tilde{\omega}_0 \exp(-L_s/l) \ll \omega \ll \tilde{\omega}_0, \quad \tilde{\omega}_0 \equiv \omega_0 \exp(A/2k_B T), \quad (23)$$

where L_s is the smallest of the sample sizes. In this case, with the logarithmic accuracy we obtain for $A \gg k_B T$,

$$\langle \delta U^2 \rangle_\omega = U_{dc}^2 \frac{\langle \delta n_2^2 \rangle_T}{\bar{n}_2} \frac{4\pi^2 l^3}{\omega} \ln^2 \left(\frac{\tilde{\omega}_0}{\omega} \right). \quad (24)$$

Thus, in the wide range of sufficiently low frequencies (23) the noise power spectrum for our system has almost $1/f$ form.

The variation $\langle \delta n_2^2 \rangle_T = V_S^{-2} (\overline{N_2^2} - \bar{N}_2^2)$ is easily found in the same way as Eq. (3),

$$\langle \delta n_2^2 \rangle_T = \frac{\bar{n}_2}{2V_s}. \quad (25)$$

Combining this with Eq. (24), we write the final expression for the spectral density of noise for $A \gg k_B T$ in the form

$$\langle \delta U^2 \rangle_\omega = U_{dc}^2 \frac{2\pi^2 l^3}{V_s \omega} \ln^2 \left(\frac{\omega_0 e^{A/2k_B T}}{\omega} \right). \quad (26)$$

V. DISCUSSION

For the further discussion, it is convenient to rewrite Eq. (26) in the form

$$\alpha = \frac{\langle \delta U^2 \rangle_\omega V_s \omega}{U_{dc}^2} = 2\pi^2 l^3 \ln^2 \left(\frac{\tilde{\omega}_0}{\omega} \right). \quad (27)$$

It is remarkable that the noise spectrum in our model has $1/f$ form up to very low frequencies. This is due to fluctuations in occupation numbers of droplets, associated with creation and annihilation of extra electron – hole pairs. This mechanism of $1/f$ noise is specific for our model and is not present in standard hopping conduction.

Let us estimate the numerical value of the parameter α , which is the standard measure of the strength of $1/f$ noise. This parameter is proportional to the third power of l . Simple estimates (analogous to that presented in the Introduction for a) reveal that, in general, l of the order or higher than a . Assuming again that the excitation energy is of the order of the Coulomb energy $A \sim e^2/a\epsilon$, taking ω_0 to be of the order of the Fermi energy inside droplets (which means $\hbar\omega_0 \sim 300K$ for $n < n_c$), and estimating the tunneling length l as being $l \gtrsim 2a \sim 20 \text{ \AA}$, we arrive to a conclusion that the parameter α is of the order $\alpha \approx 10^{-17} - 10^{-16} \text{ cm}^3$ for $T < A/k_B$ and $\omega \sim 1 \text{ Hz} - 1 \text{ MHz}$. This value of α is by several orders of magnitude higher than in usual semiconducting materials (see Ref. [15]). Such a large magnitude of the noise can be attributed to the relatively low height of the potential barrier A and to the relatively large tunneling length l . Formally, it is also related to the large value of the logarithm squared in Eq. (27).

According to Eqs. (26) and (27), the noise power and the noise parameter α are independent of the volume fraction occupied by the droplets. This result is valid in the intermediate range of n , when the droplet density is not too high and not too low. First, we assumed that the droplets are isolated point objects and the tunneling between the two droplets is not affected by a third polaron. This is only valid provided the droplet density is far from the percolation threshold, $n \ll n_c$. On the other hand, the droplet density must not be too low since the conditions $N, N_1, N_2 \gg 1$ are assumed to be met. Moreover, we neglected the possibilities of the disappearance of a droplet due to the electron tunneling, and the decay of two-electron droplets. Thus, the characteristic times of these processes should be longer than the characteristic tunneling time, and the average tunneling distance can not be too high (see Eq. (15) and the discussion below it).

The above speculations imply that the following set of inequalities should be met, $a \ll n^{-1/3} \ll l$, for formula (13) for the conductivity to be valid. In general, the tunneling length should not be much larger than the droplet radius since just the same physical parameters determine

these two characteristic distances. So, these inequalities could not be valid for real physical systems and it is of interest to consider the situation where $a, l \ll n^{-1/3}$, which is beyond the scope of our model. However, some definite conclusions concerning the magnetoresistance and the noise power can be made at present.

First, the factor $\exp(A/2k_B T)$ in the temperature dependence of the conductivity is related to the number of carriers and appears due to strong Coulomb repulsion of electrons in the droplet. It seems rather obvious that such a factor appears in the formula for the conductivity below the percolation threshold for an arbitrary relation between a and l . On the other hand, in contrast to common hopping systems, strong $1/f$ noise in our model results from fluctuations of state occupation numbers. Actually, our result for parameter α (27) only relies on the fact that $\delta\sigma/\sigma \sim \delta n_2/\bar{n}_2$, and thus the noise power is expected to have $1/f$ form below the percolation threshold for any relation between a and l . Then, we can also conclude that in our model strong $1/f$ noise will be found under the (experimentally relevant) conditions $a \sim l$.

Another important point is that we disregard direct Coulomb interaction between the droplets in comparison with the energy A . This is justified if the gas of the droplets is diluted, $n^{-1/3} \gg a$. In this respect, we recollect that in standard hopping conduction systems (doped semiconductors) the main mechanism of low-frequency noise is an exchange of electrons between the infinite cluster and nearby finite clusters. In the absence of interactions it leads to the noise power proportional to $\omega^{-\alpha}$, with the exponent α being considerably below one [19]. To explain $1/f$ noise in these systems, models involving Coulomb interactions have been proposed [20,21]. These sources of low-frequency noise are thus beyond our discussion. We also did not consider sources of noise different from resistance fluctuations. At least two other types of noise are inevitably present in the system, Nyquist-Johnson (thermal) noise, which is a consequence of the fluctuation-dissipation theorem, and shot noise due to the discrete nature of electron charge (see Refs. [16,18] for review). Both these noises are frequency independent (white) at low frequencies. The magnitudes of Nyquist-Johnson, shot, and $1/f$ noises are governed by absolutely different parameters, and we do not attempt to compare them here, noting only that at low frequencies $1/f$ noise must dominate.

In our model, we assumed that the number of droplets N is fixed and strictly equal to the number of extra electrons. In actual systems, N can also fluctuate, and this can be an additional source of noise, and of $1/f$ noise, in particular. However, this contribution depends critically on the heights of corresponding energy barriers and can vary for different systems.

As we have already mentioned, the main motivation of our work was the experimental study [6], which observed high $1/f$ noise power at high temperatures far from the metal-insulator transition. In the same exper-

iment, noise dropped to much lower levels at low temperatures in the metallic phase. This behavior of the noise power is consistent with the present model since in the metallic phase the electron tunneling contribution to the total conductivity is negligible. In the vicinity of the percolation transition the noise power increases drastically [6]. In this paper we do not attempt to describe the system of magnetic polarons close to the percolation threshold. However, we argue that the amplitude of $1/f$ noise is already large in the phase-separated regime even far from the percolation threshold.

VI. CONCLUSIONS

We emphasize that even in our oversimplified model we get a reasonable behavior of resistivity and magnetoresistance for underdoped manganites. Moreover, we have shown that in the framework of our model $1/f$ noise appears in the natural way. The phase-separation ensures a large magnitude of the noise power as compared with homogeneous materials.

Of course, a more sophisticated theory should include both the ferromagnetic structure of the droplet and the antiferromagnetic structure of the insulating matrix. This can lead us to the physics resembling that observed in the process of spin-assisted tunneling, which attracts a considerable interest nowadays (see *e.g.* Ref. [22]). The work in this direction is in progress.

ACKNOWLEDGMENTS

This work was supported by Swiss National Science Foundation, INTAS (grants 97-0963 and 97-11954), Russian Foundation for Basic Research (grants 00-02-16255 and 00-15-96570), and the Russian President Program (grant 96-15-9694). We acknowledge stimulating discussions with P. Fulde, D. Vollhardt, K. Held, P. Wölffe, A. F. Andreev, Yu. Kagan, A. S. Ioselevich, M. S. Mar'enko and D. V. Efremov. M. Yu. K. would like also to acknowledge hospitality of the Max-Planck-Institut in Dresden where a part of this work was done.

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